

# Mutual Information of RIS assisted MIMO with imperfect channel state estimation

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## Abstract

Reconfigurable intelligent surface (RIS) has appeared as one of candidates for 6G communication. RIS can reflect and modify the phase of the signal, which leads increasing the performance. One of the ways to calculate the capacity of RIS system is mutual information, which in this paper is embedded with considering the imperfect channel state estimation. The simulation result shows that mutual information of RIS performs better than conventional MIMO after getting the additional information and still less than the capacity as the maximum bits can be transmitted. Therefore, calculating the mutual information the simulation is considered to be more similar with practical implementation.

## I. Introduction

Reconfigurable intelligent surface (RIS) is a planar meta-surface consisted of reflecting elements to enhance the quality of the signal [1]. One benefit of the RIS is it can have additional information using the reflecting patterns [2]. Previously, this concept is already studied in index modulation as it can use the [3] additional bit for the activation of the resource.

However, in the previous work the perfect channel state information was assumed. Therefore, this paper studies about the imperfect channel state information as it is more similar with the real-world implementation. The simulation result is explained in Section III.

## II. System Models

In Fig. 1, the transmitter sends the downlink signals to the RIS. The phase shift signals are reconfigured in RIS and then it reflects the signal to the receiver. The received signal for  $k$ -th RIS pattern is denoted with

$$\mathbf{y} = \sqrt{\gamma} \mathbf{H}_k \mathbf{\Phi}_k \mathbf{G}_k \mathbf{x}_k + \mathbf{n} \quad (1)$$

where  $\gamma$  denotes the average received SNR. The channels from the transmitter to the RIS and from RIS to the receiver are denoted with  $\mathbf{H} = [H_1, \dots, H_{L_r}] \in \mathbb{C}^{1 \times L_r}$ , and  $\mathbf{G} = [G_1, \dots, G_{L_r}] \in \mathbb{C}^{L_r \times 1}$ , respectively. The diagonal

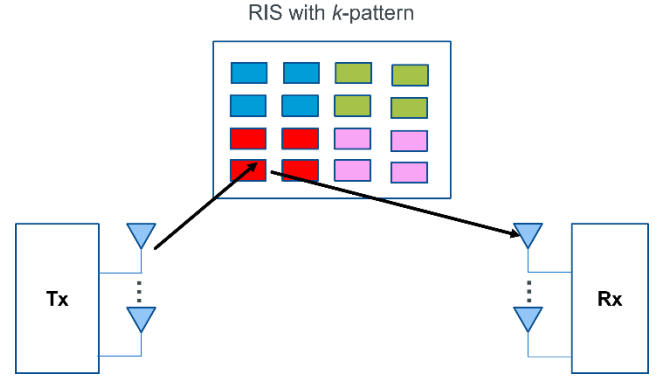


Figure 1. System model of the RIS-aided MIMO communication.

reflection matrix of the RIS is denoted with  $\mathbf{\Phi}_k = \text{diag}[\sqrt{\beta_1} e^{j\phi_1}, \dots, \sqrt{\beta_{L_r}} e^{j\phi_{L_r}}]$ . The number of RIS element is denoted with  $L_r$ .  $\mathbf{x}_k \sim \mathcal{CN}(\mathbf{0}_{N_t}, \mathbf{Q}_k)$  denotes the transmitted signal where  $\mathbf{Q}_k$  denotes the covariance matrix.

## III. Performance Analysis

In this study, the RIS assisted MIMO derived by mutual information can be written as follows

$$\begin{aligned} I(\hat{\mathbf{x}}; \mathbf{y}) &= H(\mathbf{y}) - H(\mathbf{y} | \mathbf{H}, \hat{\mathbf{x}}) \\ &= \mathbb{E}[-\log_2 f(\mathbf{y})] - N_r \log_2(\pi e), \end{aligned} \quad (2)$$

The probability density function of the received vector  $\mathbf{y}$  is given by

$$\begin{aligned}
p_{\mathbf{Y}}(\mathbf{y}) &= \int_{\tilde{\mathcal{H}}, S} p_{\tilde{\mathcal{H}}}(\tilde{\mathcal{H}}_\ell) \times p_S(S_\ell) \times p_{(\mathbf{Y}|\tilde{\mathcal{H}}, S)}(\mathbf{y} | \tilde{\mathcal{H}}_\ell, S_\ell) d\tilde{\mathcal{H}}_\ell dS_\ell \\
&= \mathbb{E}_{\tilde{\mathcal{H}}, S} \left\{ \frac{\exp\left(-\frac{\|\mathbf{y} - \tilde{\mathcal{H}}_\ell S_\ell\|_F^2}{\sigma_n^2 + \sigma_e^2 \|S_\ell\|_F^2}\right)}{\pi^{N_r} (\sigma_n^2 + \sigma_e^2 \|S_\ell\|_F^2)^{N_r}} \right\}, \quad (2)
\end{aligned}$$

and the PDF of spatial symbols can be written as

$$p_{(\mathbf{Y}|\tilde{\mathcal{H}}, S)}(\mathbf{y} | \tilde{\mathcal{H}}_\ell, S_\ell) = \frac{1}{(\pi(\sigma_n^2 + \sigma_e^2 \|S_\ell\|_F^2))^{N_r}} \exp\left(\frac{-\|\mathbf{y} - \tilde{\mathcal{H}}_\ell S_\ell\|_F^2}{\sigma_n^2 + \sigma_e^2 \|S_\ell\|_F^2}\right). \quad (3)$$

Finally, the mutual information of RIS-assisted MIMO system that consider imperfect channel estimation can be written as

$$\begin{aligned}
I(\tilde{\mathcal{H}}, S; \mathbf{Y}) &= -\mathbb{E}_S \left\{ N_r \log_2 \left( (\sigma_n^2 + \sigma_e^2 \|S_\ell\|_F^2) \exp(1) \right) \right\} \\
&\quad - \mathbb{E}_{\mathbf{Y}} \left\{ \log_2 \left( \mathbb{E}_{\tilde{\mathcal{H}}, S} \left[ \frac{\exp\left(-\frac{\|\mathbf{y} - \tilde{\mathcal{H}}_\ell S_\ell\|_F^2}{\sigma_n^2 + \sigma_e^2 \|S_\ell\|_F^2}\right)}{\pi^{N_r} (\sigma_n^2 + \sigma_e^2 \|S_\ell\|_F^2)^{N_r}} \right] \right) \right\}.
\end{aligned}$$

$$\mathcal{R}(\boldsymbol{\alpha}, \boldsymbol{\Psi}, Q) = -\int_{\mathbb{C}^{N_r}} f(\mathbf{y}) \log_2 f(\mathbf{y}) d\mathbf{y} - N_r \log_2(\pi e), \quad (3)$$

where  $\sigma_n^2$  denotes the noise variance,  $\sigma_e^2$  denotes the channel estimation error. Therefore, the lower bound equation can be derived as follows

$$\mathcal{R}^U(\boldsymbol{\alpha}, \boldsymbol{\Psi}, Q) = \sum_{k=1}^K \alpha_k (\log_2(\det \mathbf{D}_k) - \log_2(\alpha_k)),$$

where  $\mathbf{D}_k = \mathbf{I}_{N_r \times N_r} + \gamma \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^H$ .

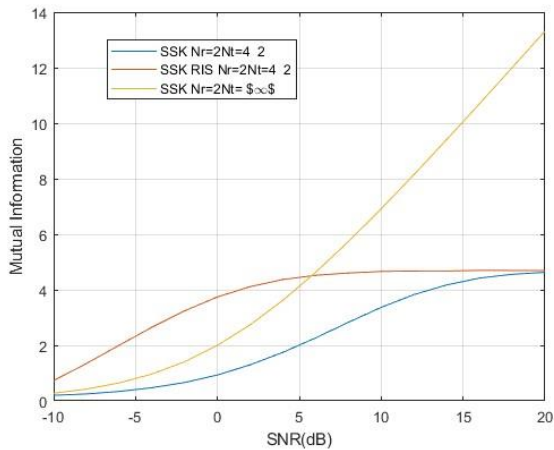


Figure 2. The mutual information of the RIS, conventional MIMO, and the channel capacity.

From the simulation result in Fig. 2, it is known that the RIS performs better than conventional MIMO terms of the mutual information in the low SNR. SSK with

infinite antenna performs the best amongst all since shows the maximum bits that can be transmitted.

#### IV. Conclusion

In this paper, the comparison between RIS and active RIS in terms of capacity is studied. The expected simulation results show that the RIS outperform the active RIS because the coverage area of RIS is larger than active RIS.

#### IV. Future Work

Since this is still an ongoing work, this study still needs a lot of improvement. For the future, this study can also consider the bit error rate parameter.

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