On the performance analysis of orbital angular momentum with multiple users

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다중 사용자를 위한 OAM 성능 분석

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Abstract

In this paper, the orbital angular momentum (OAM) channel model is dealt with focusing on multiple users. Generally, the OAM channel model includes Bessel function to show the divergence. However, the channel model can be approximated by different aspect which research requires to focus on. Therefore, modified Bessel function is used to derive channel model giving fair dirvergence effect for multiple users. The performance analysis of proposed model is simulated in terms of average capacity and outage probability.

Ⅰ. Introduction

Orbital angular momentum (OAM) in radio communiation referes to a property of waveforms that represnts a potential new degree of freedom[1]. To generate vortex waves, the uniform circular array (UCA) is recommended because of its adjustable size and supporting multiple OAM modes[2].

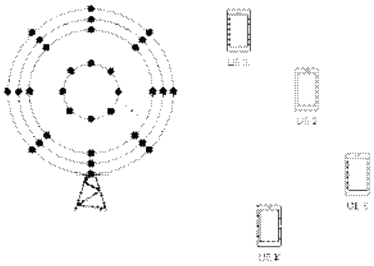

Figure 1 System model

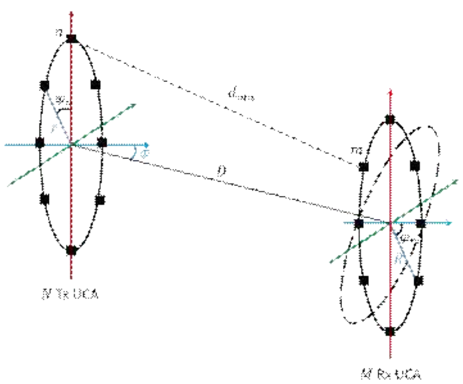

Figure 2 Geometrical transceiver

Ⅱ. System model

In this section, we suggest a system model consits of a base station (BS) and K number of OAM users (UEs) shown in Figure 1. The BS has concentric UCAs supporting multiple OAM users. We assume that every OAM user has a single UCA and the maximum number of OAM mode is determined according to the number of UCA elements. It is known that L number of OAM modes can be generated at given N number of UCA elements[ruination].

Figure 2 shows a geometrical transceiver for OAM. The number of UCA elements at the BS is identical to that of OAM users. It represents that each UCA from a BS communicates with a sinlge OAM user. The radii of transmit and receive UCAs denote r and R. The number of transmit and receive UCA elements denote n and m. The elevation angle ϕ is tilted angle between centers of transmit and receive UCAs. The center distance D is a distance between transmit and receive UCA. The element distance d_{nm} is a distance between transmit and receiver UCA elements. The element distance can be rewritten and approximated as follows

$$
\begin{aligned}
d_{nm} &= \sqrt{r^2+R^2+D^2}\sqrt{1+\frac{2(DR\sin\phi\cos\varphi_m - rR\cos(\varphi_m-\varphi_n) - Dr\sin\phi\cos\varphi_n)}{r^2+R^2+D^2}} \\
&\overset{(a)}{\approx} \sqrt{r^2+R^2+D^2}\left(1+\frac{(DR\sin\phi\cos\varphi_{\mathrm{m}} - rR\cos(\varphi_m-\varphi_n) - Dr\sin\phi\cos\varphi_n)}{r^2+R^2+D^2}\right) \\
&\overset{(b)}{\approx} D+\frac{R\sin\phi\cos\varphi_m - r\sin\phi\cos\varphi_n}{D} \qquad (1)
\end{aligned}
$$

where (a) follows the binomial approximation $\sqrt{1+x}\approx 1+x/2$ [4] and (b) follows the condition $D \gg r, R$. The multiplication of discrete Fourier transform (DFT) and inverse DFT (IDFT) is exploited with OAM line-of-sight (LoS) channel[5]. The LoS and non LoS channel model referred from [6] can be obtained using (1) as follows

$$
\begin{aligned}
h_{\mathrm{LoS}} &= \beta \frac{\lambda}{4\pi d_{nm}} e^{-j\frac{2\pi}{\lambda} d_{nm}} \\
&\approx \beta \frac{\lambda}{4\pi d_{nm}} e^{-j\frac{2\pi}{\lambda}\frac{R\sin\phi\cos\varphi_m - r\sin\phi\cos\varphi_n}{D}} \\
h_{\mathrm{NLoS}} &= \sum_{n=1}^{N} \frac{1}{\sqrt{N}} e^{-jl\varphi_n} h_{\mathrm{LoS}} \sum_{m=1}^{M} \frac{1}{\sqrt{M}} e^{jl\varphi_m} \\
&\overset{(c)}{\approx} \beta \frac{\lambda\pi NM}{D\sqrt{N}\sqrt{M}} e^{-j\frac{2\pi}{\lambda}D} \mathcal{I}\left(\frac{2\pi r\sin\phi}{\lambda D}\right) \mathcal{I}\left(\frac{2\pi R\sin\phi}{\lambda D}\right) \quad (2)
\end{aligned}
$$

Ⅲ. Performance analysis

In this section, we describe the performance of the proposed channel model in the aspect of average capacity (AC) and outage probability (OP). The AC can be expressed as follows

$$
C_k = \mathbb{E}\left\{\log_2\left(1 + \frac{\rho \sum_{l=1}^{L} |h_{\mathrm{NLoS}}|^2}{N\sigma^2}\right)\right\} \quad (3)
$$

where $E(\bullet)$ is the expectation operation and ρ is the transmit signal-to-noise ratio (SNR). L is the total number of OAM modes. N is the number of transmit UCA elements. σ is the additive Gaussian noise.

The OP is degined with cumulative distribution function (PDF) of the transmit SNR given rate threshold. The exact OP is derived from [7] can be rewritten as follows

$$
\begin{aligned}
F_k &= \{1 - \mathrm{P_r}(\gamma_{\mathrm{th}} \le 2^{C_k} - 1\} \\
&= \left\{1 - \exp\left(-\gamma_{\mathrm{th}}\left(\frac{1}{\rho\zeta_k}\right)\right)\right\} \quad (4)
\end{aligned}
$$

Ⅳ. Simulation results & Conclusion

The section presents the simulation results in terms of AC. The simulation parameters are considered as wavelength λ=0.01 where the carrier frequency is 30GHz and a coefficient related to antenna characteritics β=4π. The radii of transmit and receive antenna is given as 3[m] for UE 1, 4[m] for UE 2, 5[m] for UE 3 ($r = R$). The center distance is given as 40[m] for UE 1 , 45[m] for UE 2, 50[m] for UE 3. The number of transmit and receive UCA antenna is given as 6 ($N = M$). The total of OAM modes is given as L=[-3,3]. The elevation angles are given as 20 ° for UE 1, 10 ° for UE 2, -10 ° for UE3.

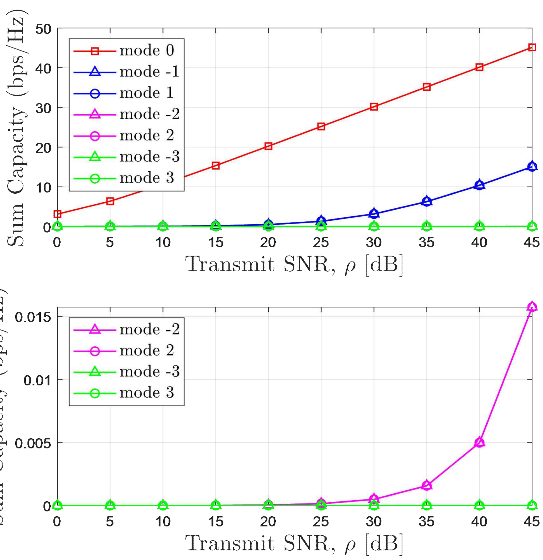

Figure 3 AC comparison

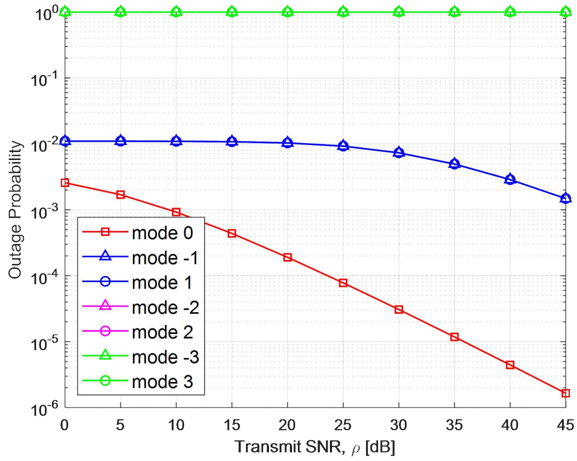

Figure 4. OP comparison

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