Gouy Phase Characteristics of Orbital Angular Momentum-Carrying Laguerre Gaussian Beam

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Laguerre Gaussian 빔을 운반하는 궤도 각운동량에 대한 Gouy phase 특징

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Abstract

This paper presents an analysis of the Gouy phase in orbital angular momentum (OAM) -carrying Laguerre-Gaussian (LG) beams. The Gouy phase, a critical parameter affecting the spatial distribution and phase structure of optical beams, is explored in the context of OAM-carrying LG beams. In this work, we present a theoretical analysis of the Gouy phase's characteristics on the LG beams with different radial modes while maintaining the identical OAM modes. The analysis provides insights into the behavior of the Gouy phase in LG beams and the findings contribute to a better understanding of the interplay between the Gouy phase and LG beam, enabling advancements in optical wireless communication systems.

Ⅰ. Introduction

The advancement of photonics and quantum optics has led to an increased interest in light beams that carry Orbital Angular Momentum (OAM). OAM-carrying beams, especially Laguerre-Gaussian (LG) beams, are unique due to their helical wavefronts and have found wide-ranging applications in areas such as optical communication, quantum information processing, and more [1]. Previous studies [2] have shown that the Gouy phase shift—an inherent phase anomaly experienced by a beam as it propagates—is a crucial characteristic within these beams that can significantly impact their behavior. While the Gouy phase shift has been extensively studied in Gaussian beams [3], its effects on OAM-carrying LG beams are less well-documented. Understanding the Gouy phase's role in LG beams could lead to more efficient and controlled manipulation of these beams and it becomes necessary.

However, when it comes to OAM-carrying LG beams, our understanding of the Gouy phase shift and its relationship with OAM modes is limited. Considering these factors, our study aims to delve into the analysis of the Gouy phase characteristics in OAM-carrying LG beams, emphasizing the relationship with radial modes.

Ⅱ. Gouy Phase in Laguerre-Gaussian Beam

The electric field of the Laguerre-Gaussian beam is given by [4]

$$
\begin{aligned}
\mathrm{U}(r,\phi,z) = \alpha & \sqrt{\frac{p!}{\pi(p+|l|)!}}\frac{1}{w(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|} \\
& \times L_P^{|l|}\left(\frac{2r^2}{w^2(z)}\right)\exp\left(-\frac{r^2}{w^2(z)}\right)\exp\left[-j\frac{z}{z_R}\left(\frac{r}{w(z)}\right)^2\right] \\
& \times \exp[-j(|l|+2p+1)\varphi(z)]\exp(-jl\phi)\exp(-jkz). \qquad (1)
\end{aligned}
$$

where (r,ϕ,z) denote cylindrical coordinates, k is the wave number, p is the radial mode index, l is the azimuthal mode index (OAM state), $w(z)$ is the beam radius, and $(|l|+2p+1)\varphi(z)$ is the Gouy phase shift with $\varphi(z)=\tan^{-1}\left(\frac{z}{z_R}\right)$. z_R is the Rayleigh range and is equal to $z_R=\frac{k\omega_0^2}{2}$, where ω_0 is the radius of the beam waist.

The Gouy phase shift is the axial phase shift that converging photons experience as they pass through the waist of the beam [4]. It is an important characteristic that affects the spatial properties and the phase structure of an OAM-LG beam can change significantly along the propagation direction and between different OAM modes due to the Gouy phase. Each OAM mode corresponds to a unique phase front structure of the beam and carries an associated phase singularity. The Gouy phase shift for an LG beam can be represented as [5]

$$\Phi(z) = \exp[-j(|l| + 2p + 1)] \tan^{-1}\left(\frac{z}{z_R}\right). \qquad (2)$$

We can see that the Gouy phase depends on the azimuthal index mode and radial mode in Eq. (2). We visualize the intensity profile and phase front of each beam in Figure 1. We can observe that LG beams' phase patterns are different due to different p values ($p = 1, 2, 3$).

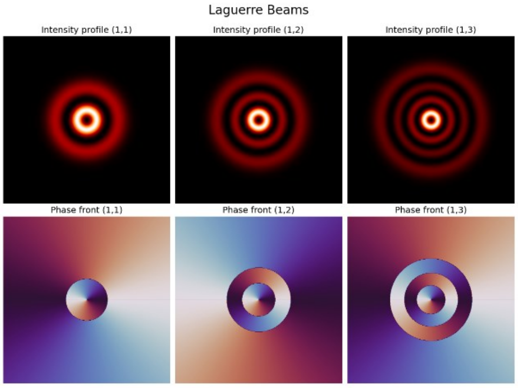

Fig. 1. Intensity profile and phase front of LG beams with different radial modes ($p = 1, 2, 3$) and identical OAM mode ($l = 1$).

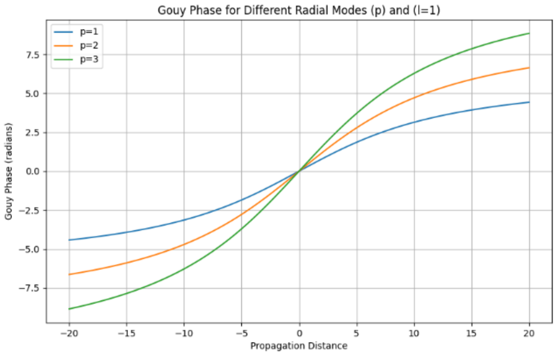

Fig. 2. Gouy phases of LG beams with different radial modes ($p = 1, 2, 3$) and ($l = 1$).

Figure 2 shows how the Gouy phase varies along the propagation direction for different radial modes (p). This relationship demonstrates how the Gouy phase is dependent on the radial mode and shows that higher values of p will have a larger phase shift compared to lower ones.

Ⅲ. Simulation Analysis

Theoretically, LG beams are orthogonal to each other if they have either a different l values and/or a different p values. It can be expressed as [6]

$$\int_0^\infty \int_0^{2\pi} LG(r,\phi,z{:}\,l_1p_1) LG^*(r,\phi,z;\,l_2p_2)\, r d\phi dr = \begin{cases} 0 & \text{if } l_1 \neq l_2 \text{ or } p_1 \neq p_2 \\ 1 & \text{if } l_1 = l_2 \text{ or } p_1 = p_2 \end{cases}. \qquad (3)$$

Based on these properties, we conducted a computational simulation to investigate the orthogonality of the LG beams, specifically focusing on variations in the radial modes (p). In Figure 3, we set the values of p to (1, 2, 3), while keeping l consistent at 1. In Figure 3, we can observe that when the p values pair up identically, i.e., (1,1) or (2,2) or (3,3), the double integral deviates from 0. However, when these p values differ from one another, the double integral is 0, which confirms the beams' orthogonality.

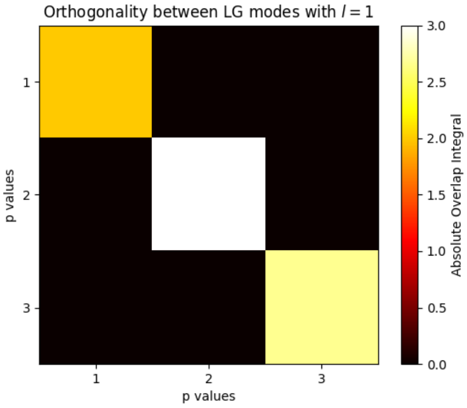

Fig. 3. Orthogonality relation between LG beams with different radial mode values ($p = 1, 2, 3$) and ($l = 1$).

IV. Conclusion

This paper has presented an analysis of the Gouy phase in OAM-carrying LG beams and its relationship with different radial modes. Then, the orthogonality with different radial modes is established by simulation. The findings reveal the potential of Gouy phase as a multiplexing parameter in high-capacity optical wireless communications.

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