Generation of Orbital Angular Momentum Modes using Laguerre-Gaussian Formula

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라구에르-가우스 공식을 이용한 궤도 각운동량 모드 생성

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Abstract

Laguerre-Gaussian (LG) beams are reciprocally orthogonal when they propagate along the same axis with distinct azimuthal states. It is recognized that the orthogonality property of the beams has the potential to improve the performance of an optical wireless communication system. In this paper, three different orbital angular momentum (OAM) modes are generated using the LG formula by simulation on MATLAB. The results show that the generated OAM modes using the LG formula meet the visualization of the OAM modes and can be used for further analysis on OAM technology for optical wireless communications.

I. Introduction

Often referred to as vortex beams, orbital angular momentum (OAM) was first reported in the optical domain in 1992 [1]. Popular for its orthogonality characteristic which provides big advantages in data multiplexing and encoding, OAM has been used to increase the system capacity in free space communications [2], ultra-high spectral efficiency optical wireless communications, and real-world long-distance application scenarios [3].

To use OAM beams in wireless communications, researchers need to generate the OAM beams which can then be multiplexed to transmit multiple data simultaneously over the same frequency. In simulation, ones can generate the OAM modes based on Laguerre-Gaussian (LG) formula using Python, MATLAB, or any other network simulators to exploit the additional degree of freedom from the OAM beams.

In this paper, we focus on the generation of multiple OAM modes using the LG formula on MATLAB. Section II describes the LG formula, Section III presents the simulation setup on MATLAB, Section IV discusses the results and conclusions are drawn in Section V.

II. Generation Method for OAM

The LG modes are a set of solutions to the paraxial wave equation in cylindrical coordinates. These modes have a helical wavefront and carry OAM. The formula for LG modes in cylindrical coordinates is expressed as [4]

$$u_{l,p}(r,\theta, z)$$
$$= \alpha \sqrt{\frac{p!}{\pi(p + |l|)!}} \frac{1}{w(z)} \left(\frac{r\sqrt{2}}{w(z)}\right)^{|l|} L_p^{|l|}\left(\frac{2r^2}{w(z)^2}\right) \exp\left(-\frac{r^2}{w(z)^2}\right)$$
$$\times \exp\left(-i\frac{z}{z_R}\left(\frac{r}{w(z)}\right)^2\right) \exp(-i(2p + |l| + 1)\psi(z))$$
$$\times \exp(-il\theta) \exp(-ikz) \qquad (1)$$

where l is the azimuthal mode number (OAM state), p is the radial mode number, r is the radial coordinate, θ is the azimuthal angle between the reference direction on the chosen plane and the line from the origin to the projection of the receiver on the plane, and k is the wavenumber which is equal to $2\pi/\lambda$. z represents the axial distance from the transmitter to the receiver. $w(z)$ is the Gaussian beam width, which can be calculated by

$$w(z) = w_0\sqrt{1 + \left(\frac{z}{2R}\right)^2} \qquad (2)$$

where w_0 is the minimum beam radius at $z = 0$. z_R is the Rayleigh range which is equal to $\frac{\pi w_0{}^2}{\lambda}$, where λ is the wavelength of the light source. $(2p + |l| + 1)\psi(z)$ is the Gouy phase shift with $\psi(z)$ equals to $\arctan(z/z_R)$. $L_p^{|l|}$ is the generalized Laguerre Polynomial of degree p and order l [6].

III. Simulation Setup

For simulation, we used the MATLAB software to generate three different OAM modes with w_0 being 1 nm, p set to 0 with the transmitter located at 3 m axial distance z from the receiver. Fig. 1 shows the OAM mode generation flow using the LG formula. A spatial grid is set up for representing the two-dimensional transverse profile of an OAM mode, with x-coordinates defined along one dimension of the spatial grid, while y-coordinates are set to be the same as the x-coordinates, creating a square grid using 'linspace' operation.
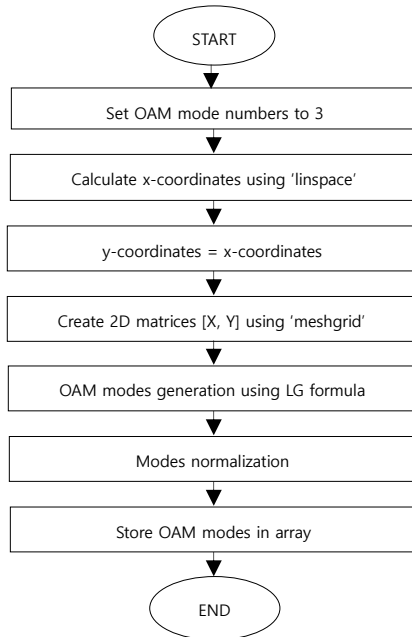
```
START
  ↓
Set OAM mode numbers to 3
  ↓
Calculate x-coordinates using 'linspace'
  ↓
y-coordinates = x-coordinates
  ↓
Create 2D matrices [X, Y] using 'meshgrid'
  ↓
OAM modes generation using LG formula
  ↓
Modes normalization
  ↓
Store OAM modes in array
  ↓
END
```

Fig. 1   OAM mode generation using the LG formula.

'meshgrid' then creates 2D matrices 'X' and 'Y' by replicating the x-coordinates and y-coordinates, respectively. The generated LG mode is then normalized before being stored in the array for each OAM mode.

IV. Simulation Results

The visualization of the generated OAM modes and their corresponding wavefront phase are shown in Fig. 2 (a) and (b) respectively. From the results, we can see that the generated OAM modes in the MATLAB simulation meet the theoretical visualization of the OAM modes in the literature [5].
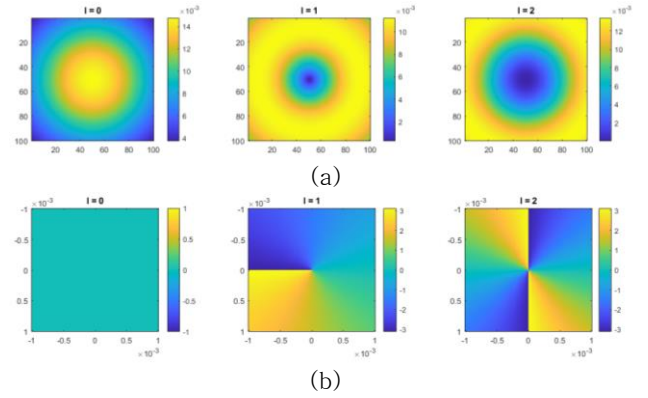

Fig. 2   The (a) intensity profile and (b) wavefront phase of the generated OAM modes with topological charge of l = 0, 1, and 2.

It also can be seen that the number of continuous phase changes of the wavefront in the azimuthal direction is determined by the value of l. The OAM mode with larger azimuthal mode number l also has larger divergence angle during the transmission which can provide larger coverage at the receiver.

V. Conclusions

In this paper, we have succeeded in generating OAM modes using LG formula by simulating the code function in MATLAB software. The generated OAM modes show significant difference in intensity profile and wavefront phase with increasing azimuthal mode number l. The LG mode terms in the LG mode formula can be used for further analysis of OAM technology to improve the performance of optical wireless communications.

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