Beamforming Optimization in Cell-Free MIMO: PCA-Quantum Learning Approach

Lia Suci Waliani*, Silvirianti†, and Soo Young Shin‡
Department of IT Convergence Engineering, Kumoh National Institute of Technology, Gumi, South Korea
e-mail: {*liasuciwaliani,†silvirianti93,‡wdragon}@kumoh.ac.kr

***Abstract*—This study proposes principal component analysis quantum learning (PCA-QL) to reduce the input dimension, addressing the limited number of qubits in quantum machine learning. As a particular application, PCA-QL is employed to optimize beamforming directions from multiple access points (APs) to multi-user equipment (UEs) in a cell-free multiple-input-multiple-output (CF-MIMO) scenario. The objective is to maximize the achievable sum rate given the optimal transmit beamforming.**

***Index Terms*—Beamforming, cell-free MIMO, principal component analysis, quantum machine learning.**

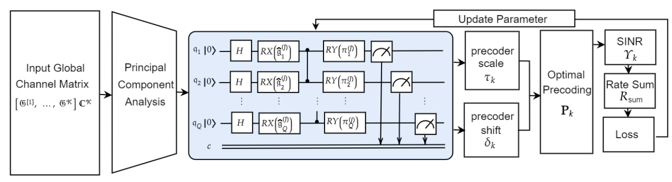

Figure 1. The proposed PCA-quantum learning for beamforming optimization.

I. Introduction

Cell-free multiple-input multiple-output (CF-MIMO) is a technology where antennas called access points (APs) are distributed to serve multi-user equipment (UEs) [1]. In CF-MIMO, each UE is not served by a dedicated single AP as in the conventional method. Instead, each UE is served by multiple APs that are randomly distributed in the service coverage area. It is worth noting that all the APs are connected to the central processing unit (CPU) through fronthaul links [2]. In order to enhance spectral efficiency, each AP has to accurately transmit the desired signal to the intended UEs. Consequently, optimally directing the beamforming becomes an important task. However, designing optimal beamforming employing conventional methods, i.e., mathematical derivation, classical machine learning, etc., can be challenging and computationally complex because of the number of optimized variables. Therefore, several studies proposed quantum machine learning based on CF-MIMO [3]. Nevertheless, to the best of the authors' knowledge, the utilization of quantum machine learning for beamforming optimization is still limited. Additionally, considering the current limited number of qubits in quantum machine learning, this study proposes principal component analysis quantum learning (PCA-QL) to reduce the input dimension. The proposed PCA-QL is employed to optimize the direction of beamforming in the CF-MIMO scenario.

II. Cell-Free MIMO System Model

This study considers a downlink CF-MIMO system consisting of $\mathcal{A}$ APs, where each AP employs N_{Tx} antennas to serve $\mathcal{K}$ UEs. In addition, both the APs and the UEs are arbitrarily distributed throughout the service coverage area. Moreover, all the APs are connected via fronthaul links to the CPU. Owing to this connection, the CPU collects the channel state information (CSI) from all APs, forming what is commonly referred to as the global channel matrix. The global channel matrix that is collected in the CPU can be expressed as $\mathfrak{G} = \left[\mathfrak{g}^{[1]}, \cdots, \mathfrak{g}^{[k]}, \cdots, \mathfrak{g}^{[\mathcal{K}]}\right] \in \mathbb{C}^{\mathcal{K}}$.

The perfect CSI is considered in this study, thus the channel coefficient between a-th AP and k-th UE can be expressed as $\mathfrak{g}_{n,k}^{[a]} = \frac{1}{\sqrt{P_{\text{path}}}} \sum_{p=1}^{P_{\text{path}}} \eta_{n,k}^{[a]} \alpha(\theta_p)$, where the $\eta_{k,a} \sim \mathcal{CN}(0, d_{n,k}^{-\chi[a]})$ denotes the complex-valued random variable for Rayleigh fading, following a normal distribution with 0 as the mean and $d_{n,k}^{-\chi[a]}$ as the variance, where χ denotes the path loss exponent. In this context, $d_{n,k}^{[a]}$ denotes the distance between the n-th antenna in the a-th AP and the k-th UE. Additionally, $\alpha(\theta_p)$ denotes the antenna response vector, which can be expressed as $\alpha(\theta_p) = \left[1, \cdots, e^{-j2\pi\theta_p\beta}\right]$, where $\theta_p \in (0, 2\pi]$ denotes the spatial direction and β can be expressed as $\beta = \left\{n - \frac{1}{2} \cdot (N_{\text{Tx}} - 1)\right\}$.

In terms of the processing method, CF-MIMO offers two schemes: centralized and decentralized. This study focuses on the centralized scheme, where a precoding matrix is performed in the CPU based on global CSI. In order to serve multiple UE, the CPU employs the zero-forcing to produce a precoding matrix. Let $\mathbf{L}_k$ be the precoding matrix for each UE. By employing zero forcing, the precoding matrix can be obtained by pseudo-inverse the channel matrix $\mathfrak{G}$. Thus, the $\mathbf{L}_k$ can be calculated as $\mathbf{L}_k = \mathfrak{G}(\mathfrak{G}^{\text{H}}\mathfrak{G})^{-1}$, where $(\cdot)^{\text{H}}$ denotes the Hermitian operation. Furthermore, according to [4], the optimized zero forcing vector can be expressed as $\mathbf{P}_k = \left(\tau_k + \mathbf{L}_k\right) + \delta_k / \left\| \left(\tau_k + \mathbf{L}_k\right) + \delta_k \right\|$, where τ_k and δ_k denote the scale and shift factor, respectively. Furthermore, these optimized factors can be obtained from quantum machine learning.

It is noteworthy that not all APs can serve the UEs due to an out-of-range condition, as a result of the distance between the AP and UE that exceeds the maximum threshold.

Therefore, the APs that serve the UEs are grouped as $\mathcal{A}_s$, where $\mathcal{A}_s = \{1, \cdots, A_s\}$. Meanwhile, the APs that do not serve the UEs are considered as interference and grouped as $\mathcal{A}_i$, where $\mathcal{A}_i = \{1, \cdots, A_i\}$. The received signal-to-interference-plus-noise ratio (SINR) then can be expressed as $\Upsilon_k = \frac{\sum_{a_s \in \mathcal{A}_s} \xi \left| \mathfrak{G}_{k,a_s}^{\mathrm{T}} \mathbf{P}_k^{[a_s]} \right|^2}{\xi \sum_{a_i \in \mathcal{A}_i, a_i \notin \mathcal{A}_s} b_{k,a_i} \left| \mathfrak{G}_{k,a_i}^{\mathrm{T}} \mathbf{P}_k^{[a_i]} \right|^2 + 1}$, where ξ denotes the transmitted signal-to-noise power ratio which can be calculated as $\xi = {}^{O_{\mathrm{Tx}}}/{\sigma^2}$ and $b_{n,k_i} \in (0, 1]$ denotes the interference factor. This study assumes that all the APs have a similar transmit power O_{Tx}. The achievable rate for the k-th user can be expressed as $R_{\text{sum}} = \sum_{k=1}^{\mathcal{K}} \log_2(1 + \Upsilon_k)$.

Problem Formulation As mentioned earlier, the objective function is to maximize the achievable sum rate given the precoder matrix which can be formulated as follows:

$$\max_{\mathbf{P}} \quad R_{\text{sum}} \tag{1a}$$

$$\text{s.t.} \quad C_1 : R_{\text{sum}} \geq R_{\text{min}}, \tag{1b}$$

$$C_2 : ||P_k||^2 \leq 1, \forall k \in \{1, \ldots, \mathcal{K}\} \tag{1c}$$

III. PCA-Quantum Learning for Beamforming Optimization

The PCA process for reducing input dimension consists of several steps that can be specifically described as follows. Let $\mathfrak{G} \in \mathbb{C}^{\mathcal{A} \times \mathcal{K}}$ which is the global channel matrix that the CPU receives from all APs, be the input training data. First, to reduce the dimension of $\mathfrak{G} \in \mathbb{C}^{\mathcal{A} \times \mathcal{K}}$, we calculate the covariance matrix from the centered data of the channel matrix that can be expressed as $\Sigma = {}^1/_{\mathcal{A}} \bar{\mathfrak{G}}^{\mathrm{H}} \bar{\mathfrak{G}}$, where $\bar{\mathfrak{G}}$ denotes the centered global channel matrix. Second, perform eigenvalue decomposition to reveal the principal components; the eigenvalues represent the total number of variance captured by each component, while the eigenvectors define their directions. Third, by sorting the eigenvalues and eigenvectors in descending order, highlight the principal components that capture the most significant patterns in the data. Thus, the projection matrix $\mathbf{M}$ can be expressed by selecting the top k eigenvectors as columns, and the matrix is written as $\mathbf{M} = [\lambda_1, \cdots, \lambda_k]$. Finally, the reduced input training data can be expressed as $\hat{\mathfrak{G}} = \bar{\mathfrak{G}} \times \mathbf{M}$.

Subsequently, the $\hat{\mathfrak{G}} = [\hat{\mathfrak{g}}_1, \cdots, \mathfrak{g}_k]$ become inputs of the quantum machine learning. Herein, the number of the inputs is equal to the number of qubits Q. where each qubit is placed into the superposition state employing the Hadamard operator, denoted by $\mathbf{H}$, which can be expressed as $\mathcal{U}_{(\text{sup})} \triangleq \bigotimes_{q=1}^{Q_{\text{qubit}}} \mathbf{H}(|\mathfrak{g}_q\rangle)$. Thereafter, feedforward training process of quantum machine learning denoted by $\mathcal{U}_{\text{PCA-QL}}$, can be expressed as follows $\mathcal{U}_{\text{PCA-QL}} \triangleq \big(\bigotimes_{j=1}^{J_{\text{layer}}} \bigotimes_{q=1}^{Q_{\text{qubit}}} \mathbf{RY}\big(\pi_q^{[j]}\big)\big(\prod_{j=1}^{J_{\text{layer}}} \prod_{q=1}^{Q_{\text{qubit}}} \mathbf{CX}\big(\mathfrak{q}_q^{[j]} | \mathfrak{q}_{q-1}^{[j]}\big) \otimes \ldots \otimes \mathbf{CX}\big(\mathfrak{q}_{Q_{\text{qubit}}}^{[j]} | \mathfrak{q}_{Q_{\text{qubit}}-1}^{[j]}\big)\big) \mathbf{RX}\big(\hat{\mathfrak{g}}_q^{[j]}\big)\big)$, where the N_{qubit} denotes the number of required qubits. In the last layer, the quantum measurement is performed to obtain the classical values that can be expressed as $\mathcal{M}_{\text{PCA-QL}} = \langle 0 | \mathcal{U}_{\text{PCA-QL}}(\vartheta)^{\dagger} H \mathcal{U}_{\text{PCA-QL}}(\vartheta) | 0 \rangle$, where in order to mitigate the noise in the quantum computing, the measurement is repeated S_{shot} times. Finally, the classical values as the outputs from quantum machine learning can be expressed as $\mathcal{U}_{\text{decode}} = \frac{1}{S_{\text{shot}}} \sum_{s=1}^{S_{\text{shot}}} \mathcal{M}_{\text{PCA-QL}}^{[s]}$. The loss is computed on the classical computer, as the PCA-QL employs unsupervised learning then the loss function can be expressed as $\mathbb{L} = -R_{\text{sum}}$. Subsequently, based on the loss, the gradient can be calculated by employing parameter-shift rules [5] which can be expressed as $\nabla \mathbb{L}_{\text{PCA-QL}}(\vartheta) = \frac{\mathbb{L}_{\text{PCA-QL}}(\vartheta + \epsilon) - \mathbb{L}_{\text{PCA-QL}}(\vartheta - \epsilon)}{2 \sin(\epsilon)}$, where the shifting phase is denoted by $\epsilon \in [0, \pi]$. At the last, parameter update can be expressed as $\omega^* = \omega - \alpha \nabla \mathbb{L}_{\text{PCA-QL}}(\vartheta)$, where α denotes the learning rate.

IV. Conclusion

In this study, a principal component analysis quantum learning (PCA-QL) was proposed to reduce the input dimension to address the issue of the current limited number of qubits in quantum machine learning. As a particular application, the PCA-QL was employed to optimize the beamforming direction to obtain the maximum achievable sum rate in a cell-free MIMO scenario.

Acknowledgments

This research was supported by the MSIT(Ministry of Science and ICT), Korea, under the ICAN(ICT Challenge and Advanced Network of HRD) program(IITP-2022-RS-2022-00156394) supervised by the IITP(Institute of Information & Communications Technology Planning & Evaluation) and Institute of Information & communications Technology Planning & Evaluation (IITP) grant funded by the Korea government(MSIT) (No. 2021-0-02120, Research on Integration of Federated and Transfer learning between 6G base stations exploiting Quantum Neural Networks).

References

[1] H. Hojatian, J. Nadal, J.-F. Frigon, and F. Leduc-Primeau, “Decentralized beamforming for cell-free massive mimo with unsupervised learning,” *IEEE Commun. Lett.*, vol. 26, no. 5, pp. 1042–1046, May, 2022.

[2] M. Zaher, T. Demir, E. Björnson, and M. Petrova, “Learning-based downlink power allocation in cell-free massive mimo systems,” *IEEE Trans. on Wireless Commun.*, vol. 22, no. 1, pp. 174–188, Jan., 2023.

[3] B. Narottama and T. Q. Duong, “Quantum neural networks for optimal resource allocation in cell-free mimo systems,” in *GLOBECOM 2022 - 2022 IEEE Global Commun. Conf.*, Dec., 2022, pp. 2444–2449.

[4] Silvirianti and S. Y. Shin, “Sub-connected hybrid precoding and trajectory optimization using deep reinforcement learning for energy-efficient millimeter-wave uav communications,” *IEEE Wireless Commun. Lett.*, vol. 12, no. 9, pp. 1642–1646, Sept., 2023.

[5] D. Wierichs, J. Izaac, C. Wang, and C. Y.-Y. Lin, “General parameter-shift rules for quantum gradients,” *Quantum*, vol. 6, p. 677, 2022.