Kinematics and Control A Three-wheeled Mobile Robot with Omni-directional Wheels

Ma Soe Soe Htay
Mechatronic Engineering Department
Mandalay Technological University
Mandalay, Myanmar
soesoehtaymc@gmail.com

Khaing Win Phyu
Mechatronic Engineering Department
Mandalay Technological University
Mandalay, Myanmar
khaingwinphyu@gmail.com

Su Yadanar
Mechatronic Engineering Department
Mandalay Technological University
Mandalay, Myanmar
suyadanarmtu@gmail.com

Wut Yi Win
Mechatronic Engineering Department
Mandalay Technological University
Mandalay, Myanmar
wutyiwin@gmail.com

***Abstract*—This paper describes the kinematics and control a three-wheeled omni-directional mobile robot. The kinematic model is derived using the geometric relationship that governs the motion of the system. The focus is on the connection between control parameters and the system's behavior in the state space. The established kinematic model is validated by the accuracy of simulation results. A prototype of a three-wheeled omni-directional mobile robot has been constructed and designed.**

Keywords—omni-directional, kinematic model, geometric relationship.

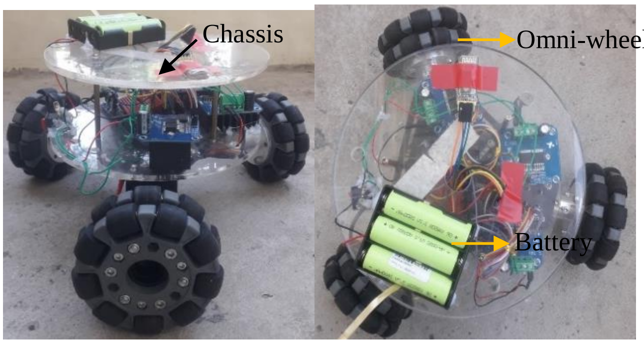
I. Introduction

In the modern automated industry, the mobile robot is more flexible and has the ability to perform more tasks effectively. An omni-directional wheeled mobile robot is currently a unique class of mobile robots. Numerous discussions have already taken place about omni-directional mobile robots. The kinetic performance of robots with different numbers of wheels varies. Therefore, many scholars have studied robots with three-wheels, four-wheels, and six-wheels[1][2][3]. Changing the position and azimuth of wheels will result in a distinct kinetic performance. Thus, the robot that has a continuously variable transmission is also being studied[4]. Correlational analysis is conducted on kinematic and dynamic modeling[5][6][7]. Most of the research is focused on the model that has a fixed arrangement of wheels. The aim of this paper was to propose an omni-directional mobile robot base that has three independent driving wheels that are equally spaced at 120 [deg]. The kinematic controller is proven to be stable asymptotically (exponentially).

As follows is the organization of the paper. Three-wheeled drive and its design are introduced in Section II. Three-wheeled omni-directional mobile robots' kinematics are calculated in section III. Section IV presents the simulation and measurement of the velocity of a mobile robot. The conclusion is given in section V.

II. Three-Wheeled Drive

The maneuverability of a wheel mobile robot depends on the wheels and drives used. Due to its rich maneuverability and simple control, the mobile robot used a three-wheeled drive in this paper. An omni-wheel is a wheel that is typically augmented with rollers on its outer circumference. These rollers spin freely about axes in the plane of the wheel and tangential to the wheel's outer circumference, and they allow sideways sliding while the wheel drives forward or backward without slip in that direction. To achieve an arbitrary three-dimensional chassis velocity, an omni-directional mobile robot must have at least three wheels, as each wheel has only one motor. Attached to the shaft of the dc motor are three omni-wheels that act as the driving wheels. At intervals of 120 [deg], those were fixed symmetrically.



(a) Front view (b)Top view

Fig. 1. Designed of a three-wheeled omni-directional mobile robot (a) front view (b) top view.

III. Kinematics of Robotic Platform

In this section, the kinematic model of a three-wheeled omni-directional mobile robot is derived. Table (1) shows the parameter description in kinematic model. Fig. 2 depicts its kinematic diagram that is used to find the kinematic model of the robot.

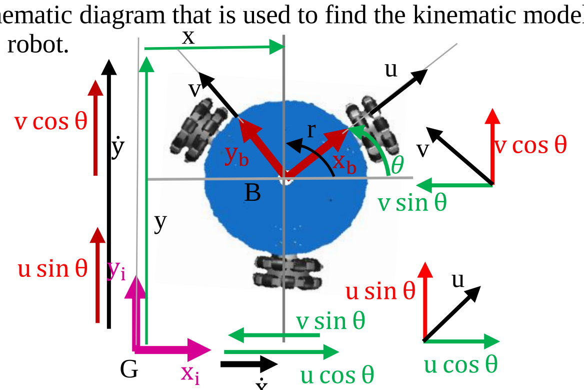


Fig. 2. Kinematic models of a three-wheeled omni-directional chassis are represented by global coordinate system, frame G and robot coordinate system, frame B.

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}u\cos\theta - v\sin\theta\\ u\sin\theta + v\cos\theta\\ r\end{bmatrix} = \begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}u\\ v\\ r\end{bmatrix} \quad (1)$$

$$\dot{\eta} = J(\theta)\xi \quad (2)$$

$$\begin{bmatrix} u \\ v \\ r \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \tag{3}$$

$$\xi = J^{-1}(\theta)\dot{\eta} \tag{4}$$

It describes the relation between the velocity input commands (ξ) and derivatives of generalized coordinates($\dot{\eta}$) $J(\theta)$is the Jacobian (or velocity transformation) matrix.

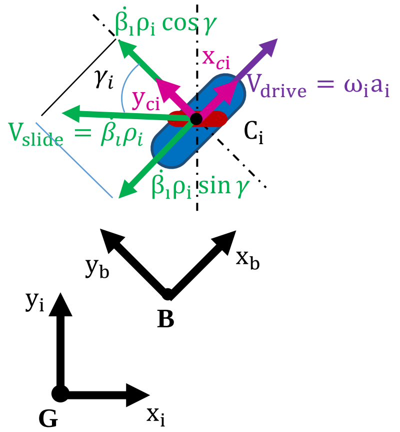


Fig. 3. Coordinate systems of one omni-directional wheel: (global coordinate system, frame G), (robot coordinate system, frame B) and (wheel coordinate system, frame C_i)

$$\omega_i = \begin{bmatrix} \frac{1}{a_i} & \frac{1}{a_i}\tan\gamma_i \end{bmatrix} \begin{bmatrix} \dot{x}_{ci} \\ \dot{y}_{ci} \end{bmatrix} \tag{5}$$

It describes the relation between driving direction and free sliding direction.

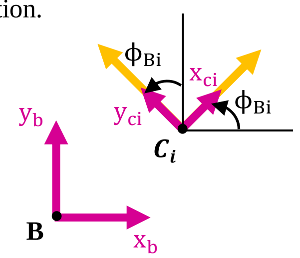


Fig. 4. x_{ci} and y_{ci} are rotating of robot coordinate frame B to wheel coordinate system frame C_i and one omini-directional wheel center in x, y direction.

$$B_{V_{ci}} = \begin{bmatrix} \cos\phi_{Bi} & -\sin\phi_{Bi} \\ \sin\phi_{Bi} & \cos\phi_{Bi} \end{bmatrix} \begin{bmatrix} \dot{x}_{ci} \\ \dot{y}_{ci} \end{bmatrix} \tag{6}$$

It describes the relation between the robot frame and the wheel frame.

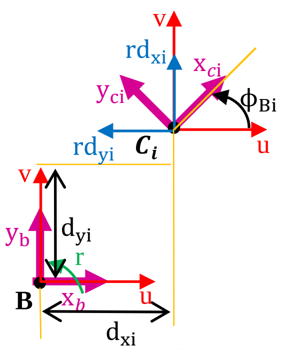


Fig. 5. Rotation and transformation of one omni-directional's robot coordinate frame B to wheel coordinate frame C_i.

$$u - rd_{yi} = \dot{x}_{ci}\cos\phi_{Bi} - \dot{y}_{ci}\sin\phi_{Bi} \tag{7}$$

$$v + rd_{xi} = \dot{x}_{ci}\sin\phi_{Bi} + \dot{y}_{ci}\cos\phi_{Bi} \tag{8}$$

$$\begin{bmatrix} 1 & 0 & -d_{yi} \\ 0 & 1 & d_{xi} \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} = \begin{bmatrix} \cos\phi_{Bi} & -\sin\phi_{Bi} \\ \sin\phi_{Bi} & \cos\phi_{Bi} \end{bmatrix} \begin{bmatrix} \dot{x}_{ci} \\ \dot{y}_{ci} \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} \dot{x}_{ci} \\ \dot{y}_{ci} \end{bmatrix} = \begin{bmatrix} \cos\phi_{Bi} & \sin\phi_{Bi} \\ -\sin\phi_{Bi} & \cos\phi_{Bi} \end{bmatrix} \begin{bmatrix} 1 & 0 & -d_{yi} \\ 0 & 1 & d_{xi} \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} \tag{10}$$

By substituting equation (10) in equation (5)

$$\omega_i = \begin{bmatrix} \frac{1}{a_i} & \frac{1}{a_i}\tan\gamma_i \end{bmatrix} \begin{bmatrix} \cos\phi_{Bi} & \sin\phi_{Bi} \\ -\sin\phi_{Bi} & \cos\phi_{Bi} \end{bmatrix} \begin{bmatrix} 1 & 0 & -d_{yi} \\ 0 & 1 & d_{xi} \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} \tag{11}$$

By substituting equation (3) in equation (11)

$$\omega_i = \begin{bmatrix} \frac{1}{a_i} & \frac{1}{a_i}\tan\gamma_i \end{bmatrix} \begin{bmatrix} \cos\phi_{Bi} & \sin\phi_{Bi} \\ -\sin\phi_{Bi} & \cos\phi_{Bi} \end{bmatrix} \begin{bmatrix} 1 & 0 & -d_{yi} \\ 0 & 1 & d_{xi} \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \tag{12}$$

TABLE I

Parameters description in kinematics model

Symbol	Description	Units
x_i	forward displacement of the mobile robot w.r.t. G	[m]
y_i	lateral displacement of the mobile robot w.r.t. G	[m]
θ_i	angular displacement of the mobile robot w.r.t. G	[deg]
u	forward velocity of the mobile robot w.r.t. B	[m/s]
v	lateral velocity of the mobile robot w.r.t. B	[m/s]
r	angular velocity of the mobile robot w.r.t. B.	[m/s]
ω_i	angular velocity of the i^{th} wheel	[deg/s]
a_i	radius of the i^{th} wheel	[m]
$\dot{\beta}_i$	angular velocity of the passive roller	[deg/s]
ρ_i	radius of the passive roller	[m]
γ_i	angle between roller axis to the x_{ci} axis	[deg]
ϕ_{Bi}	angle between the vehicle frame B and the wheel frame C_i.	[deg]
d_{xi}, d_{yi}	position coordinates of C_i with reference to B.	[m]

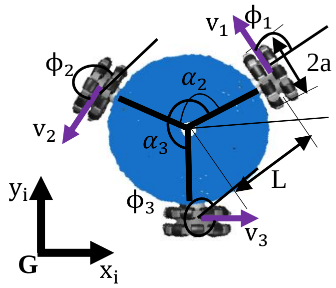


Fig. 6. Configuration of three-wheeled omni-directional mobile robot with respect to global coordinate system, frame G.

These are the parameter of the three-wheeled omni-directional mobile robot

$\alpha_i = [0° \quad 120° \quad 240°]$, $d_{xi} = L\cos\alpha_i$, $d_{yi} = L\sin\alpha_i$

$a_1 = a_2 = a_3 = a$, $\phi_{B1} = 90°$, $\phi_{B2} = 210°$, $\phi_{B3} = 330°$

$d_{x1} = L\cos(0°), = L, d_{x2} = L\cos(120°) = -\frac{L}{2}$,

$d_{x3} = L\cos(240°) = -\frac{L}{2}$, $d_{y1} = L\sin(0°) = 0$,

$d_{y2} = L\sin(120°) = \frac{\sqrt{3}L}{2}$, $d_{y3} = L\sin(240°) = -\frac{\sqrt{3}L}{2}$

$\gamma_1 = \gamma_2 = \gamma_3 = 0°$

By substituting the robot parameter in equation (11)

$$\omega_1 = \frac{1}{a}(v + Lr) \tag{13}$$

$$\omega_2 = \frac{1}{2a}(-\sqrt{3}u - v + 2Lr) \tag{14}$$

$$\omega_3 = \frac{1}{2a}(\sqrt{3}u - v + 2Lr) \tag{15}$$

It can be written in vector-matrix form.

$$\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} = \frac{1}{a}\begin{bmatrix}0 & 1 & L\\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & L\\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & L\end{bmatrix}\begin{bmatrix}u\\ v\\ r\end{bmatrix} \tag{16}$$

$$\begin{bmatrix}u\\ v\\ r\end{bmatrix} = a\begin{bmatrix}0 & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3}\\ \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3}\\ \frac{1}{3L} & \frac{1}{3L} & \frac{1}{3L}\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} \tag{17}$$

$$\xi = W\omega \tag{18}$$

It describes the kinematic model base on wheel configuration.

The kinematic control is described to achieve asymptotically (exponentially) stable, $t \rightarrow \infty, \eta \rightarrow \eta_d$ or in other words, $t \rightarrow \infty, \tilde{\eta} \rightarrow 0$, $\tilde{\eta}(t) \rightarrow e^{-\lambda t}, \lambda > 0$, where $\tilde{\eta} = \eta_d - \eta$. Choosing the control input based on the computed velocity control as : $\xi = J^{-1}(\eta)[\dot{\eta}_d(t) + \lambda\tilde{\eta}(t)]$. Based on this control input vector that can find the individual wheel velocities of a three-wheeled omni-directional mobile robot, as: $\omega = W^{+}\xi$. Where, η is actual position, η_d is desire position, $\dot{\eta}_d$ is desire velocities, $J(\eta), J^{-1}(\eta)$ is Jacobian matrix, W^{+} is pseudo inverse matrix and ξ is control input.

IV. Simulation

In this section, the simulation is carried out to verify the accuracy of the derived kinematic model of three-wheeled omni-directional mobile robot. The coordinate (x_i, y_i) of three-wheeled omni-directional mobile robot in moving processing are shown in Fig. 7. It can be seen from this figure that the movement trajectory is very close to the target trajectory. The tracking error between the movement trajectory and the target trajectory is shown in Fig. 8.

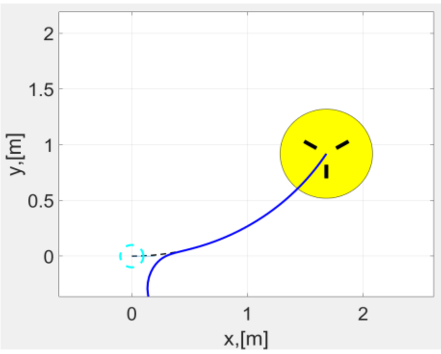


Fig. 7. Simulation result of three-wheeled omni-directional mobile robot coordinate system.

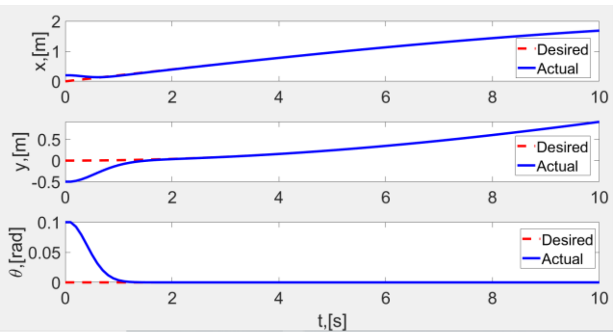


Fig. 8. Simulation of vehicle responses error tracking result using MATLAB.

Two position parameters, x_i and y_i and the orientation angle θ_i are given in the subfigure of Fig.8. The initial position of (x_i ,y_i, θ_i) is set to (0.2, -0.5, 0.1) and target position is set to (2*sin (0.1*t(i)), 2-2*cos (0.1*t(i)), 0). The kinematic controller can steer the omni-directional mobile robot to the target quickly and steadily. The velocity input throughout the whole motion process is shown in Fig. 9.

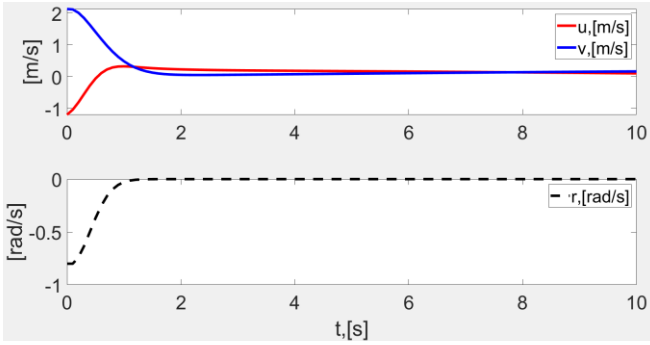


Fig. 9. Control velocities of three-wheeled omni-directional mobile robot.

V. Conclusions

This paper has been discussed and simulated of kinematic three omni-directional wheel mobile robot. With the kinematic diagram of the robot's wheels arrangement equally spaced at 120 [deg] from one another. The kinematic model has been derived using the geometrical relationship that govern the motion of the system. Simulations results have shown that the kinematics controller is capable of accomplishing desired regulation and trajectory tracking requirement.

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